About the Origin of Cities

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Cities form a hierarchical system that shows some regularity which is stressed by geographers

This begs the following two questions

(i) What are the conditions for cities and urban hierarchies to emerge in a featureless space?

(ii) How can this be derived from the interplay between local and global interactions among agents pursuing their own interests?

The term central place is used as shorthand for peaks in the spatial distribution of agents

How may

(i) a pattern of equally-spaced central places having the same size and (ii) a hierarchy of central places emerge from a symmetry-breaking process?

Many different approaches

(i) Central place theory (geography and regional science)

(ii) Spatial competition theory (Hsu, 2012)

(iii) Economic geography (Fujita et al.,1999)

(iv) Urban economics (see below)

(v) Spatial quantitative economics

Urban economics

1. Systems of cities (from Henderson, 1974 to Davis and Dingel, 2019).

Cities are like "floating islands": trade costs between cities are zero or cities are autarkies

2. The formation of employment centers within a city (from Ogawa and Fujita, 1980, to Lucas and Rossi-Hansberg, 2002)

Does geometry matter?

Beckmann (1976): land use and social interactions give rise to a bell-shaped distribution over a compact interval. But the peak melts down when this interval becomes arbitrarily wide

Mossay and Picard (2011): individuals are distributed over a circle and the equilibrium involves multiple outcomes, having any odd number of identical and evenly spaced cities

Our approach: the simplest possible seamless and unbounded space, i.e., the real line

but then, the spatial equilibria are the solutions of integral equations

We assume:

(i) a local congestion effect, which depends on the local population density, and

(ii) an exponential decay function, which depends on the whole distribution of individuals over space

Both effects are reduced forms consistent with different narratives

The interplay between the two external effects take on board the main forces at work in the formation of urban systems

You’ll hate my model …

You'll hate my model because

there is no explicit market

I. Agglomeration in a homogeneous world

The model

- A continuum of identical agents.
- We consider the class of population densities, denoted by *n(x)*, defined by non-negative, piecewise continuous, and bounded mappings from R to R_+ whose mean is given and defined by

$$\bar{n} \equiv \lim_{b \to \infty} \frac{1}{2b} \int_{-b}^{b} n(x)\,\mathrm{d}x < \infty.$$

- Preferences

$$u(x) = E(x) - \alpha n(x)$$

local congestion (competition for land): $\alpha > 0$

- Spatial externality

$$E(x) = \int_{\mathbb{R}} \exp\{-\beta|x-y|\} n(y) \mathrm{d}y$$

- Exponential gravity equation

$$n(x) n(y) \exp\{-\beta|x-y|\}$$

A high (low) value of α means that people focus more (less) on what is going on at the local level than on the various benefits generated by the interactions with the rest of the population

β is an inverse measure of the efficiency of the spatial interaction technology. Examples include communication devices and transportation means

The population distribution $n^*(x)$ is a spatial equilibrium if

(i) all individuals enjoy the same utility level

$$E^*(x) - \alpha n^*(x) = u^*$$

when $n^*(x) > 0$

(ii) $n^*(x) = 0$ when

$$E^*(x) - \alpha n^*(x) < u^*$$

(iii) the mean of $n^*(x)$ is equal to $\bar{n} > 0$

Some properties

1. The uniform distribution is always a spatial equilibrium

2. A spatial equilibrium involves no unpopulated areas

3. Any spatial equilibrium is continuous over R

- Any solution to

$$n(x) = \frac{1}{\alpha}\left[\int_{\mathbb{R}} \exp\{-\beta|x-y|\}n(y)\mathrm{d}y - u^*\right]$$

is a spatial equilibrium

The above equation is a Fredholm integral equation of the second kind: the unknown function *n(x)* appears inside and outside the integral

$$\phi \equiv \frac{2}{\alpha\beta} > 0$$

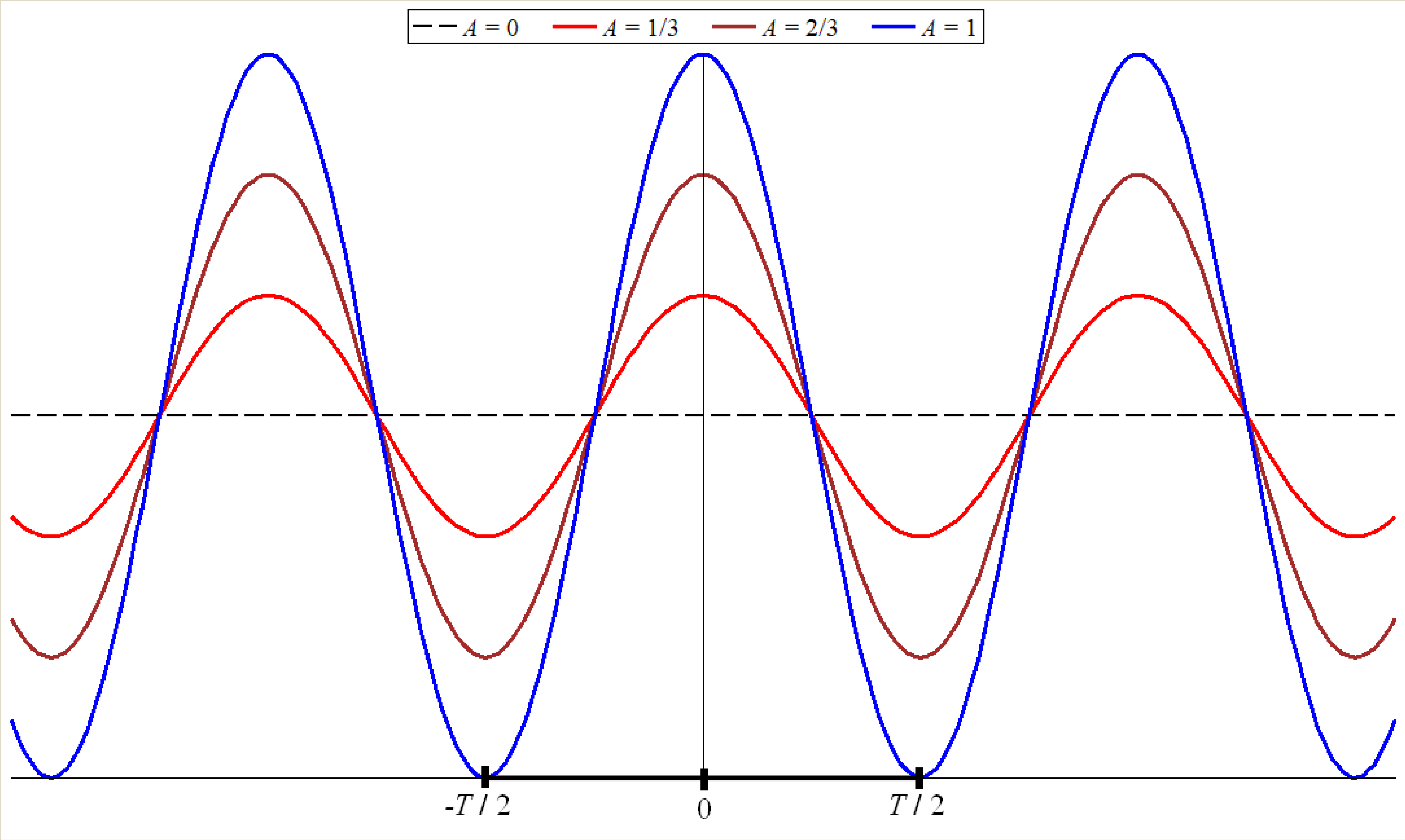
Proposition 1. The uniform distribution of individuals is the unique spatial equilibrium if and only if $\phi < 1$. Furthermore, this equilibrium is stable.

Proposition 2. If $\phi \geq 1$,

then each spatial equilibrium is given by the population distribution

$$n^*(x) = \bar{n} + A\sin\left(\beta\sqrt{\phi - 1}\ x\right)$$

where A is an arbitrary positive constant such that $A \leq \bar{n}$


A = 0
A = 1/3
A = 2/3
A = 1
-T / 2
0
T / 2

- The equilibrium population distribution is maximized at the central places given by

$$x_{\mathrm{max}}^{k} = \pm\left(\frac{1}{2} + 2k\right)\frac{\pi}{\beta}\sqrt{\frac{1}{\phi-1}} \qquad k = 0, 1, \ldots$$

- It is minimized at

$$x_{\mathrm{min}}^{k} = \pm\left(\frac{3}{2} + 2k\right)\frac{\pi}{\beta}\sqrt{\frac{1}{\phi-1}} \qquad k = 0, 1, \ldots$$

The spatial equilibrium displays periodically distributed oscillations with the period

$$T = \frac{2\pi}{\beta}\sqrt{\frac{1}{\phi - 1}}$$

The economic space is formed by a succession of identical cities whose population size is equal to $\bar{n}T$,

while the distance between cities is T

(i) The distance between two central places increases when local congestion matters less to individuals (α decreases)

(ii) T is U-shaped in β: *the packing of activities is the densest, and the size of cities the smallest, for intermediate degree of efficiency of the communication or transportation technology* (the bell-shaped curve of spatial development in economic geography)

There exists a continuum of equilibria

Proposition 3. If $\phi > 1$, then the spatial equilibria with $A < \bar{n}$ are unstable. Furthermore,

$$n^*(x) = \bar{n}\left[1 + \sin\left(\beta\sqrt{\phi - 1}\ x\right)\right]$$

Is stable.

The population at a central place $x_{\max}^{k}$ is equal to $2\bar{n}$

The population decreases around every central place

Changing the parameters α or β affects the population and physical sizes of cities, but not the population peak

What about welfare

Proposition 4. For all values of ϕ, the equilibrium utility level is given by

$$u^* = \bar{n}\left(\frac{2}{\beta} - \alpha\right) = \bar{n}\alpha(\phi - 1)$$

Any shock that leads individuals to agglomerate is welfare-enhancing

Back to the market

$$U(x,s) = E(x) - \alpha/s + z$$

where s is the size of the land plot and z the consumption of the numéraire

Budget constraint: $z + sR(x) = Y$

Indirect utility: $V(x) = E(x) - \alpha n(x) - Y$

II. Hierarchy in a heterogeneous world

The model

Two distinct populations

$$u_j(x) = \gamma_{jj} E_{jj}(x) + \gamma_{jk} E_{jk}(x) - \alpha_{jj} n_j(x) - \alpha_{jk} n_k(x)$$

$$E_{jk}(x) \equiv \int_{-\infty}^{\infty} \exp\{-\beta_{jk}|x-y|\} n_k(y) \mathrm{d}y$$

Example: agents are consumers (type 1) and firms (type 2)

- Consumers are attracted to places where the number of firms is high because there are more opportunities ($\alpha_{12} < 0$)
- Firms are attracted by places where consumers are numerous because the expected volume of business is larger ($\alpha_{21} < 0$)

- Consumers are repulsed by places where the population is high because they dislike congestion ($\alpha_{11} > 0$)
- Firms dislike places where the number of firms is high because competition is tougher ($\alpha_{22} > 0$)

A system of two Fredholm integral equations

$$\tilde{n}_j(x) = \int_{\mathbb{R}} g_{jj}(x,y)\tilde{n}_j(y)\mathrm{d}y + \int_{\mathbb{R}} g_{jk}(x,y)\tilde{n}_k(y)\mathrm{d}y$$

A system of two Fredholm integral equations

$$\tilde{n}_j(x) = \int_{\mathbb{R}} g_{jj}(x,y)\tilde{n}_j(y)\mathrm{d}y + \int_{\mathbb{R}} g_{jk}(x,y)\tilde{n}_k(y)\mathrm{d}y$$

The spatial equilibrium depends on the nature and intensity of the local and global interactions within and between populations

The welfare level of *j*-type individuals

$$u_j^* = \left(2\frac{\gamma_{jj}}{\beta_{jj}} - \alpha_{jj}\right)\bar{n}_j + \left(2\frac{\gamma_{jk}}{\beta_{jk}} - \alpha_{jk}\right)\bar{n}_k$$

Both population means affect (positively or negatively) the welfare level in each population

Equilibrium in an asymmetric world

The values of the parameters are whatever you want

Let **D** be a (4×4)-matrix involving all the parameters, but independent of x

Proposition 5. (i) If **D** has no strictly negative real eigenvalue, then the spatial equilibrium is unique and given by the uniform distributions

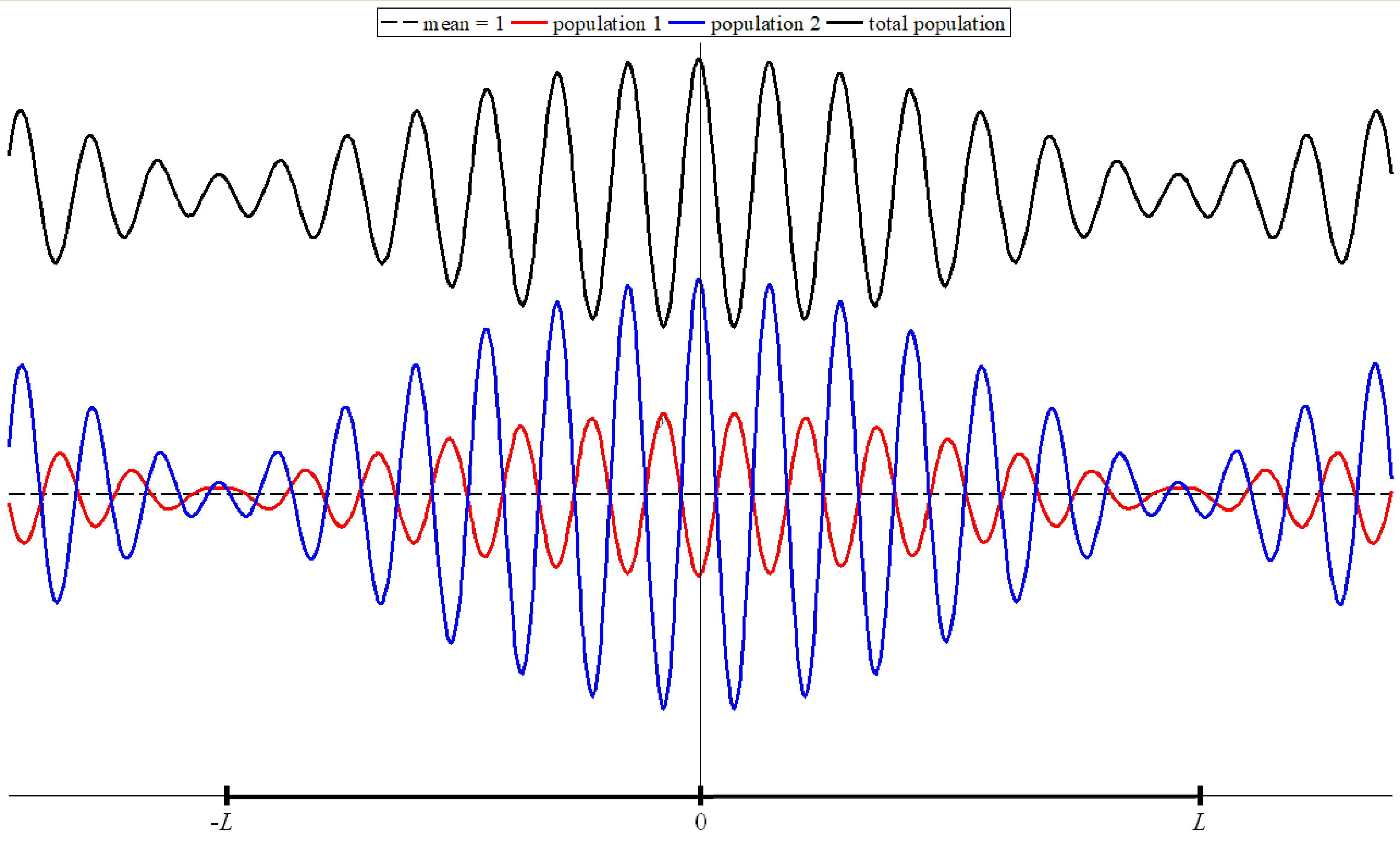
(ii) if **D** has one strictly negative real eigenvalue λ, then the non-uniform spatial equilibria are periodic and given by

$$n_i^*(x) = \bar{n}_i + A_i \sin(\sqrt{-\lambda}\ x)$$

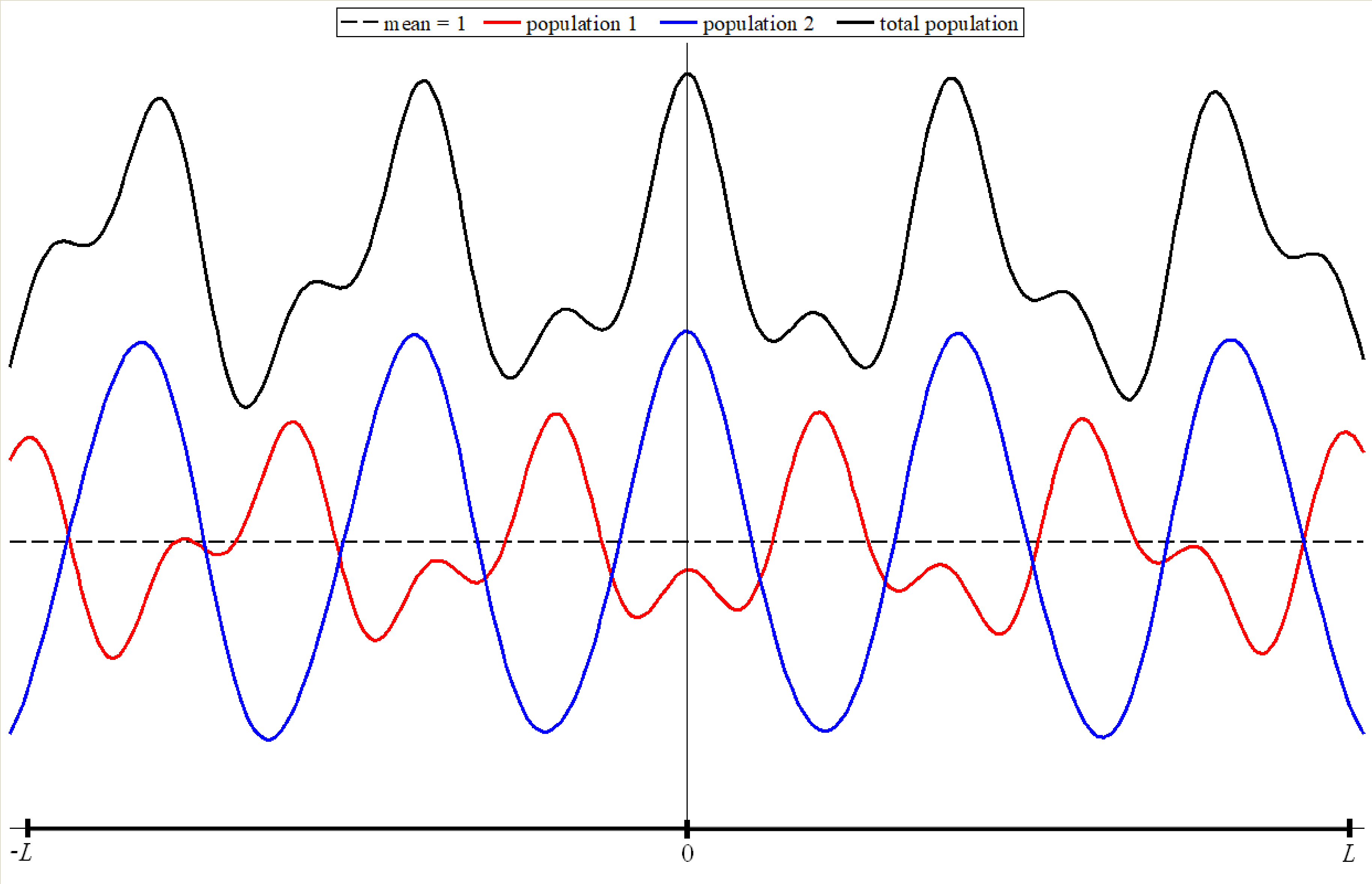
(iii) if **D** has at least two strictly negative real eigenvalues λ_1 and $\lambda_2 \neq 2k\pi\lambda_1$, k=1,2, …, then the spatial equilibria are given by

$$n_i^*(x) = \bar{n}_i + A_{i1}\sin(\sqrt{-\lambda_1}\;x) + A_{i2}\sin(\sqrt{-\lambda_2}\;x)$$

These equilibria involve central places having different sizes



mean = 1
population 1
population 2
total population
-L
0
L


mean = 1
population 1
population 2
total population
-L
0
L

Equilibrium in a symmetric world

The values of the parameters are as follows:

$$\alpha \equiv \alpha_{11} = \alpha_{22} > 0 \qquad \alpha_{12} = \alpha_{21} = 0$$

$$\gamma_{11} = \gamma_{22} = 1 \qquad \gamma_{12} = \gamma_{21} = \gamma \in (0, 1)$$

$$\beta \equiv \beta_{jk} > 0$$

Proposition 6. (i) If $\phi \leq 1/(1+\gamma)$, there exists a unique spatial equilibrium. This equilibrium is such that both populations are uniformly distributed

(ii) if $1/(1+\gamma) < \phi \leq 1/(1-\gamma)$, there exist non-uniform equilibria involving central places that have the same size

(iii) if $\phi > 1/(1-\gamma)$, there exist non-uniform equilibria with central places of different sizes

A hierarchy emerges under either weak spatial impedance, that is, low transportation or communication costs, or low congestion costs, or both (akin to Proposition 2)

Conclusion

(i) Our setting accounts for a rich set of urban patterns

(ii) It provides a background for two of the main features of classical central place theory: (a) a pattern of equidistant and identical settlements and (b) a hierarchy of cities

(iii) Our analysis also shows that there is no reason to expect the urban system to obey the rigid, pyramidal structure assumed by Christaller and Lösch. The urban system may involve two or several large cities at the top of the urban hierarchy

Work in progress

Spatial externality kernel

$$E(x) = \int_{\mathbb{R}} g(x-y)n(y)\mathrm{d}y > 0$$

A1: $g(\cdot) \in L_1(R)$

$$\mathcal{G} \equiv \int_{-\infty}^{\infty} g(z)\mathrm{d}z < \infty$$

A2: $g(\cdot)$ is weakly decreasing over R_+

A3: $g(\cdot)$ is symmetric w.r.t. the origin

Examples of kernels

$$g(z) \equiv \begin{cases} 1, & |z| \leq R, \\ 0, & |z| > R, \end{cases}$$

$$g(z) \equiv \begin{cases} 1 - |z|/b, & |z| \leq b, \\ 0, & |z| > b, \end{cases}$$

$$g(z) \equiv \exp\{-\beta|z|\}$$

$$g(z) \equiv \exp\{-z^2/2\sigma^2\}$$

Proposition 7. The uniform distribution is the unique spatial equilibrium if and only if $\mathcal{G} \leq \alpha$. Furthermore, this equilibrium is stable.

Consider the Fourier transform of the spatial externality kernel

$$\hat{g}(\xi) \equiv \int_{-\infty}^{\infty} \exp\{i\xi x\} g(x) \mathrm{d}x$$

Proposition 8. If

(i) $\mathcal{G} > \alpha$ and

(ii) $\widehat{g}(\xi)$ is single-peaked,

then all non-uniform equilibria are given by

$$n^*(x) = \bar{n} + A\cos(ax)$$

where A > 0 is a constant and a > 0 the unique solution to $\widehat{g}(a) = \alpha$

Proposition 9. If

(i) $\mathcal{G} > \alpha$ and

(ii) $\widehat{g}(\xi)$ is not single-peaked,

then all non-uniform equilibria are given by

$$n^*(x) = \bar{n} + \sum_{k=1}^{m} [A_k \cos(a_k x) + B_k \sin(a_k x)]$$

Thank you for your attention